

# **GUM Uncertainty Analysis with no Greek Letters**

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## **Abstract**

Is it possible to create a GUM compliant uncertainty analysis without an advanced course in statistics and calculus? Is it possible to have uncertainties documented so that someone other than the author can read it? Yes!

All that is needed is a tool. With this paper is provided a comprehensive Microsoft Excel\* customized spreadsheet [1] that will do all of the required math and documentation for you. You can follow the examples on your own computer as we discuss the topics that matter to you. Uncertainty analysis can stop being a pain and start being an important information resource for controlling your laboratory.

\* Microsoft Excel is a registered trademark of Microsoft Corp.

## **1 Introduction**

Agilent Technologies, Inc. has an extensive ISO-17025 program currently being executed. This aggressive program is most visible in the customer sales and support services. Calibration centers and manufacturing final test facilities are worldwide. The level of maturity and experience is widely divergent. This program is unifying certificates, measurement reports, services, marketing programs, quality systems and how uncertainties are documented.

Uncertainties need to be calculated and documented. The strategy was to find a single tool that accomplishes both, and meets the criteria that...

- 1) Is easy to understand.
- 2) Implements GUM [2] methodology for calculating uncertainties.
- 3) Does not compromise real issues and is mathematically accurate.
- 4) Flexible.
- 5) Applicable in 90% of all uncertainty analysis situations.

This paper describes that tool and its use.

## 2 Choice of tool

The 17025 Metrology Leadership Team evaluated six tools. Two of these tools were Microsoft Excel spreadsheets. All six met the 5 criteria and tested accurate. The final choice was the non-commercial Excel Spreadsheet currently in use at the Loveland Standards Lab. The reasons that tipped the scale were...

- 1) Very Flexible -- is implemented with common spreadsheet usage.
- 2) Minimal use of Macros (2). One to calculate sensitivities, the other copies data in the Report.
- 3) The Microsoft Office tools are a basic skills requirement at Agilent.
- 4) Parameters can be assigned and the measurement equation written using a familiar venue.
- 5) It is supportable and is currently in use within Agilent.

This paper has in fact, been re-written several times as this tool has played an increasingly important and successful role in the 17025 program. Other choices (for example a commercial tool) can be a better choice for the reader. You may, for example, have only one test executive that you use and it has a tool. You may not want to support, and develop training materials for an internal tool. However, to get the same benefits that have been experienced and is described below, it is imperative to settle on a single tool that meets the criteria.

## 3 How to Follow the Examples in This Paper

You will need to download the Microsoft Excel Workbook (.xls) [1] in order to read the remainder of this paper. Once you have a copy on your computer, open it and allow macros to be activated if so prompted. This example spreadsheet is a hybrid of two actual experiences applying this tool. The names (equipment and people) have been changed to protect the guilty.

In this paper you will be asked to SEE, DO, LEARN, and RULE(s) just as users are asked to observe in training materials. DO and SEE what is requested. It is best to follow the sequence as presented by this paper and do not skip sections. References to tabs will be in quotes, i.e. TAB "Rev". Addresses and Ranges will be in double brackets [[E24]].

DO: Open a copy of Workbook [1].

SEE: Four Tabs

Tab "Rev": Revision History of the document (not involved in this exercise)

Tab "DUT, Vs, Zo": Information about the DUT, and two variables defined by and unique to this example. (Source Voltage and Load Resistance)

Tab "En": A detailed Report about the En contributor (Repeatability)

Tab "Er": A detailed Report about the Er contributor (Reproducibility)

Tab "Unc": GUM style uncertainty worksheet.

Tab "Report" A summary page when the uncertainty analysis applies to more than one parameter. (Unc can be parameter driven)

DO: Browse the four Tabs.

LEARN: Vs and Zo are names invented by the user. En and Er are names reserved for the illustrated purpose.

RULE: The author of an uncertainty analysis will identify the contributors to the analysis and name them. The values associated with those names will be accessed through links (preferably

in the same workbook). References Tabs will bear the name of the contributor and provide the information required. DUT information will include pertinent information from the procedure associated with this analysis. The reference Tabs do not have to follow formatted look provided in the example, but must as a minimum contain the same information.

#### **4 Introducing Tad and Derek**

Tad and Derek are preparing their first 17025 uncertainty analysis using this new tool. The procedure is an existing procedure for the calibration of a power meter. All references and required information will be contained in this one Workbook. You are the Senior Metrologist supporting their procedure. They have called you for support. You gave them instruction (over the phone) and are following along with a copy of their effort so far.

DO: Click TAB "Report"

SEE: ET90-0293: Uncertainties Report (Old Way)

Tad, "We did the analysis as you instructed, but we don't get the same results that we had using the old method. (Old Way). We are required to have a minimum TAR of 2.0 by our Quality Manual. We have had a lot of trouble getting this DUT to pass the 100kW scale, could this be why? "

Yes, the GUM analysis could be correct. The difference could be because during the last teleconference Tad was asked to provide information about an additional Contributor Er. But that is not the only difference. We can take the effect of Er out and see how much effect it has.

DO: Click TAB "Report"

Unhide Columns [[B]] and [[C]]

Insert 0.001 in [[K7]], [[K8]], and [[K9]]

Insert X in [[C7]], [[C8]] and [[C9]]

Press Cntrl-Shift-R

SEE: The Report will update indicating the effect of near zero Er Variation.

LEARN: The Er Variation is causing you to not meet the goal of TAR=2.0 but the GUM has shown that the (Old Way) also understated the effect of uncertainty in the applied standards.

#### **5 Contributors**

Tad, "How do we know if we have included all of the contributors."

LEARN: There are some very useful references containing lists of error sources.

UKAS, Pub: LAB12 [3] has a list of ways to get type B estimations (7.1.3) and a list of common sources of uncertainty (6.1). However, we will not use them to identify contributors.

RULE: Every Uncertainty Analysis will include Er plus a contribution for each value considered in the measurement. (see Measurement Equation section 5)

Derik, "Isn't Er like double counting? Don't the specification limits in the applied standards Vs and Zo include that variation?"

LEARN: Excellent observation. It does.

Er is required because it includes all of those variations due to the applied standards, plus the most important variation -- the variation you cause in the way that you implement your procedure -- intended or unintended.

First we want to know how dominant compared to the other contributions  $E_r$  is found to be. Obtaining values for  $E_r$  can be very expensive and time consuming. On the first estimate of  $E_r$  no more than 5 samples should be taken.

**RULES:** Handling  $E_r$ .

- 1) Do 5 complete calibrations according to the documented procedure. ( $df=4$ )
- 2) Do one each day of one workweek.
- 3) Allow variability that is permitted by the procedure (eg. Different operators, cords, shift, etc)
- 4) If  $E_r$  is found to be the Dominant contributor, but an increase in the degrees of freedom would meet your goal, then proceed to Analyze  $E_n$ .
- 5) If  $E_r$  is found to be the Dominant contributor, but you are not sure why, then proceed to Analyze  $E_n$ , or partition  $E_r$  to reduce the double counting (Consult your Senior Metrologist)

## 6 Measurement Equation.

The Measurement Equation is an Excel expression that you enter into the Cell named "Equation". Derek, "The old way let us RSS the Specification Limits of the applied standards, isn't that enough? I've tried to read the GUM [2], but I don't get it. If we follow the rules for this spreadsheet (New Way), will that be enough for GUM?"

**LEARN:** Yes, as long as we meet the GUM assumptions that allow the spreadsheet.

- Assumptions:**
- 1) Linear: Contributions can be modeled as  $a_1*C_1+a_2*C_2+ \dots$
  - 2) No cross-correlation between the contributors
  - 3) No dominant contributor has different from Normal distribution.
  - 4) All Contributions are unbiased.

Derek, "Oh, great ☺"

The use of this spread sheet will allow us to make simpler rules. The measurement equation can be thought of as the formula that was used in the software, or procedure worksheet to calculate the actual value of a parameter being applied to the DUT. In other words, the value that you expect to read on the DUT if there is no error.

**RULES:** Measurement equation.

- 1) A measurement equation is in fact a function. It is an Excel expression that you enter into the Cell named "Equation".
- 2) The only input variable cells referenced in that function will reside between the cell named "Nom", [[I9]] and that one which is holding the function named "Equation" [[I14]].
- 3) The Value placed in the cells [[I10:I13]] will be the nominal (or what is considered to be no error value)
- 4) The units for the values in the cells [[I10:I13]], must be noted in [[K10:K13]], as well as the units of the result in [[K14]]. Refer to these units as you write the measurement equation.
- 5) No input to the function should be dependent upon the value of another input. If so, then re-write the function.
- 6) Do not declare a distribution to be anything other than Normal, without a good reason. When in doubt call it Normal.

Derek, "Oh, great ☺, I can understand these rules."

Tad, "Didn't you fail to address assumption 1?"

Yes, but that is very rarely violated. The reason is that we are working in a calibration lab where

the deviations from nominal are exceedingly small. If however, adjustments that you make before taking a reading are very sensitive (eg. Tuning near resonance, or nulling some value to zero) then call a Senior Metrologist for assistance.

Derek, "I thought that all Type B data was to be treated as a Rectangle Distribution."

That is a common practice, but it leads to miss-understandings. NIST Guidelines [4] state, "The rectangular distribution is a reasonable default model in the absence of any other information. But if it is known that values of the quantity in question near the center of the limits are more likely than values close to the limits, a triangular or a normal distribution may be a better model." Another good reference is Howard. Castrup's paper. [5]. You can assume that equipment that is adjusted to the center of its limits during calibration, is Normal.

Rule: If you believe that you have a dominant contributor that is not normal, then this approach is not appropriate and you need to consult a Senior Metrologist, who will use an entirely different approach more appropriate to your situation, such as the Hetman FFT [6]

## 7 Analysis of Rn data

DO: Re-load the example to reset the changes.

Now we will look at the work that Derek and Tad did gathering Type A data for evaluating En.

DO: Click TAB "En".

Tad, "We only modified the software to collect 100kW data. As can be seen in the graph, it is important to wait 60 seconds as the Full Load Model x-678 Manual suggests and I verified that it is implemented in the software. I used the standard deviation of the data last 60 seconds. I verified that the software takes and reports one reading rather than the average of several."

DO: Click TAB "Er".

LEARN:

Er includes En. In order to find out how large is Er without En, we need to use the formula in [[I47]]. Of course when En is very small, this formula has no effect on Er.

DO: Click [[I45]]

Create a link, =En!E46

Click TAB "Report"

Unhide Columns [[B]] and [[C]]

Type X in [[C9]] (and press ENTER)

Press Cntrl-Shift-R

SEE: The Report will update indicating the effect of En Variation.

Tad, "WOW, this meets the  $TAR > 2$  requirement, could this be correct? Are we done?"

LEARN:

The small number of degrees of freedom associated with the Er contribution, coupled with the fact that it was the dominant contributor, caused the analysis to "fatten" the result. We could have gotten the same result by taking much more Er data so that we would have more degrees of freedom in [[H12]] of TAB "Unc". We decided to take a chance, with En because we can take many readings at one experiment. The same number of degrees of freedom would have taken 60 days to collect with Er. We were lucky because the En component of Er accounts for most of

that uncertainty.

Tad, "I said, are we done???"

## 8 Something is Wrong

No Tad, we are not done. Remember what Derek said about double counting? It is very common that Er turns out to be large. However when it is larger than any other contributor, look for opportunities to improve the procedure.

Derek, "Wait everybody! I was checking the manual for the active load, and it says that you need to average at least 6 readings in not less than 5 seconds".

Derek seems to have found the answer. The specs for Zo in [[J11]] of TAB "Unc" are not valid for the procedure as written.

### ACTION:

Derek and Tad will modify the procedure software and the uncertainty analysis to reflect the taking of 6 data points and averaging them. Because we have En data that we expect will be reduced by the square root of 6, can you the reader predict the expected savings?

## 9 Summary.

Institutionalization of a simple to use tool, and a common vocabulary will result in the improvement of many existing procedures. Hidden in this example was the partitioning of variation by frequency, ANOVA, Welch-Satterthwaite treatment of t-distribution effective degrees of freedom for sampled data, units conversion, and sensitivity analysis.

## References

1. Robert Brown, "GUM Uncertainty Analysis with no Greek Letters", NCSLI August 2003. Available with example spreadsheet: <http://metrologyforum.tm.agilent.com/uncalc.shtml>
2. ISO, 'Guide to the Expression of Uncertainty in Measurement', prepared by ISO Technical Advisory Group 4 (TAG 4), Working Group 3 (WG 3), October 1993.
3. UKAS, "The Expression of Uncertainty in Testing", Publication ref: LAB12. October 2000.
4. NIST, " Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results", pp 4.6 available: <http://physics.nist.gov/Pubs/guidelines/sec4.html>
5. Howard. Castrup, Ph.D., "A Critique of the Uniform Distribution" available: [http://www.isgmax.com/Articles\\_Papers/Uniform%20Distribution%20Usage.pdf](http://www.isgmax.com/Articles_Papers/Uniform%20Distribution%20Usage.pdf)
6. A. Hetman, M.J. Korczynski, "Calculation of Expanded Uncertainty", 2nd On-Line Workshop on Tools for Education in Measurement, June 1-15 2002.