

## Relativistic Time Shifts for ‘Flying Clocks’

The notes in this annex are derived from ITU Recommendation ITU-R TF.1010-1 concerning “Relativistic Effects in a Co-ordinate Time System in the Vicinity of the Earth”.

The co-ordinate time accumulated when transferring time from point  $P$  to point  $Q$  by means of a portable clock in a rotating reference frame is:

$$\mathbf{Dt} = 2\mathbf{wA}_E/c^2 + \mathbf{Integral}([1 - \mathbf{gh}/c^2 + \mathbf{v}^2/2c^2]\mathbf{ds})$$

where

$\mathbf{w}$  : angular velocity of the rotation of the earth

$\mathbf{A}_E$  : equatorial projection of the area swept out by the vector  $\underline{r}$ , from the centre of the Earth to the portable clock. It is measured in an Earth-fixed co-ordinate system, and is positive when the path of the clock projected onto the equatorial plane moves eastwards.

$\mathbf{c}$  : speed of light

$\mathbf{g}$  : acceleration due to gravity evaluated at the geoid (i.e. mean sea level)

$\mathbf{h}$  : height of the portable clock above the geoid (i.e. mean sea level)

$\mathbf{v}$  : velocity of the clock with respect to the ground

$\mathbf{ds}$  : increment of time accumulated on the portable clock

and

$$\mathbf{w} = 7.292 \times 10^{-5} \text{ rad/s}$$

$$\mathbf{c} = 2.99792458 \times 10^8 \text{ ms}^{-1}$$

$$\mathbf{g} : 9.780 + 0.052\sin^2(\phi) \text{ ms}^{-2}, \text{ and } \phi \text{ is the geographical latitude}$$

It is convenient distinguish three components that make up  $\mathbf{Dt}$ :

(i) Height term :  $-\mathbf{gh}/c^2$

(ii) Speed term :  $\mathbf{v}^2/2c^2$

(iii) Sagnac term :  $2\mathbf{wA}_E/c^2$

For clarity, a clock above mean sea level will run at a faster rate than an equivalent clock on the ground.

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## **Estimates of Relativistic Time Shifts for Proposed London-Shanghai-London 'Flying Clock' Trip**

The time shifts for the London-Shanghai-London trip are estimated here using the input parameters listed below.

### **Locations**

London: latitude 51° N, longitude 0° E

Shanghai: latitude 31° N, longitude 121.5° E

### **Flight Information**

Distance = 9250 km (from locations assuming a great circle route)

Duration (OUT) = 10 hours (estimate from flight time tables)

Duration (RTN) = 11 ½ hours (estimate from flight time tables)

Altitude = 10000 m (estimate)

Speed (OUT) = 257 m/s (estimate based on distance and duration figures)

Speed (RTN) = 223 m/s (estimate based on distance and duration figures)

### **Relativistic time shifts expected to be seen on the Portable clock**

JOURNEY	SPEED term (ns)	HEIGHT term (ns)	SAGNAC term* (ns)	TOTAL (ns)
London- Shanghai	-13.2	+39.3	-40	-13.9
Shanghai- London	-11.5	+45.1	+40	+73.9
Round-trip	-24.7	+84.4	0	+59.7

\* NOTE: As a first approximation, it has been assumed that the same flight path is used on the out bound and return flights. The Sagnac effects on each leg of the journey then cancel. The figures quoted in the table above are an average of a path following the 31°N parallel (Sagnac = 51 ns) and the 51°N parallel (Sagnac = 28 ns).

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